IENAC12TE/ Econometrics 1 Final Examination

Date: Wednesday 28 May 2014.

Time allowed: 2 hours (08:00-10:00).

Answer all questions briefly.

Show all computations (including relevant critical values).

You are allowed all lecture handouts and notes, but no textbooks.

An English–French dictionary is allowed, as is a scientific calculator.

Question 1 is for 100 marks.

You are given a cross-sectional dataset on 1080 house sales in Louisiana (United States) during the year 2005. The variables are PRICE (house sale price in U.S. dollars), SQFT (surface area of house, in square feet; 1 foot ≈ 0.3048 metres), BEDROOMS (number of bedrooms), BATHS (number of bathrooms), AGE (age of house in years), OCCUPANCY (discrete variable taking value = 1 if the seller currently occupies the house, = 2 if the house is vacant, = 3 if the seller is currently renting the house to a tenant), POOL (discrete variable taking value = 1 if the house has a swimming pool, = 0 if there is no pool), STYLE (discrete variable that denotes the 'architectural style' of the house, taking value = 1 for 'traditional', = 2 for 'townhouse', = 3 for 'ranch', = 4 for 'New Orleans', = 5 for 'mobile home', = 6 for 'garden', = 7 for 'French', = 8 for 'cottage', = 9 for 'contemporary', = 10 for 'colonial', = 11 for 'Acadian (Canadian colonial)'), FIREPLACE (discrete variable taking value = 1 if the house has a fireplace, = 0 otherwise), WATERFRONT (discrete variable taking value = 1 if house next to river, = 0 otherwise), DOM (number of days that house has been for sale (on the market)). **This is used in Question 1.** Throughout, $\log(\cdot)$ refers to the natural logarithm; and (e.g.) "9.12E − 05" = 0.0000912.

1 Question 1

- This question uses the house sales data (**refer to Figures 1-30**).

  (a) Perform a careful first analysis of the variables, and explain your findings. Refer to the descriptive statistics, and scatterplots of the variables and their logarithms.

  (20 marks)

  (b) Test (i) whether the skewness of PRICE is equal to 0, against the two-sided alternative, at the 95% level, using the test statistic:

$$\frac{n}{6}\mathrm{SK}^2 \sim \chi^2(1),$$

  where n is the sample size, SK is the sample skewness, and the statistic follows the chi-squared distribution asymptotically under the null hypothesis. Test (ii) whether the kurtosis of DOM is equal to 3, against the two-sided alternative, at the 99% level, using the test statistic:

$$\frac{n}{24}(\mathrm{KT}-3)^2 \sim \chi^2(1),$$

  where KT is the sample kurtosis, and the statistic follows the chi-squared distribution asymptotically under the null hypothesis. Finally, test (iii) the normality of log(SQFT), against the two-sided alternative, at the 90% level, using the Jarque-Bera test.

  (10 marks)

(c) The following price function has been estimated using least squares (`EQ01`):

$$\begin{aligned}\log(\text{PRICE}_i) &= \beta_0 + \beta_1 \log(\text{SQFT}_i) + \beta_2 \text{BEDROOMS}_i + \beta_3 \text{BATHS}_i \\ &+ \beta_4 \text{AGE}_i + \beta_5 \text{“OCCUPANCY} = 1\text{”}_i + \beta_6 \text{POOL}_i \\ &+ \beta_7 \text{“STYLE} = 1\text{”}_i + \beta_8 \text{FIREPLACE}_i \\ &+ \beta_9 \text{WATERFRONT}_i + \beta_{10} \text{DOM}_i + u_i,\end{aligned}$$

where $i = 1, 2, \ldots, n$ indexes the observation. Note that “OCCUPANCY $= 1$” is a discrete variable that takes the value $= 1$ when OCCUPANCY$= 1$ (the seller currently occupies the house), and takes the value $= 0$ otherwise. The discrete variable “STYLE $= 1$” takes the value $= 1$ when STYLE$= 1$ (‘traditional’ house style), and takes the value $= 0$ otherwise.

(i) What are the numerical values of the estimated coefficients? (ii) Which of the estimated coefficients correspond to elasticities? Interpret those coefficients. (iii) What are the numerical values of the estimated standard errors? (iv) Are the explanatory variables individually or jointly significant? (v) What is the value of R^2? (vi) What is the value of $\overline{R}^2$? (vii) What is the value of $\widehat{\sigma}^2$? (viii). Are the signs and magnitudes of the estimated coefficients reasonable? Explain. (ix) Do the estimated residuals from `EQ01` appear to be normal? (x) Do the scatterplots of the fitted residuals from `EQ01` (‘RESID_EQ01’) against log(SQFT), and against DOM, cast doubt on any of the classical assumptions of linear regression? Explain.

(20 marks)

(d) Test $H_0 : \beta_1 = 0.5$ against $H_1 : \beta_1 > 0.5$, at the 95% level, in `EQ01`. Interpret.

(10 marks)

(e) In `EQ03`, what is the estimated elasticity of SQFT with respect to PRICE? For what values of SQFT is this elasticity positive?

(10 marks)

(f) Compare the regression output for `EQ01`, `EQ02`, `EQ03`, and `EQ04`. Which model would you use in practice? Justify your answer. What additional analysis could you perform to confirm this?

(10 marks)

(g) Write a one page summary of your empirical project (that you presented recently in class). Discuss your data collection, your methodology (model-building), and your main conclusions. What were the limitations of your research, and how could you improve your study to deal with them?

(20 marks)

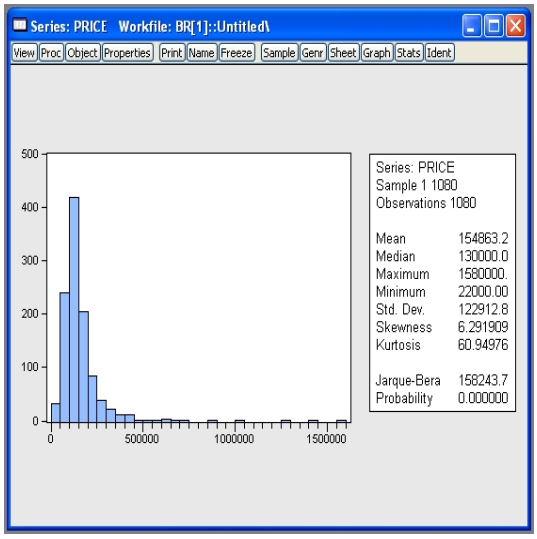


Figure 1: Statistics on PRICE.

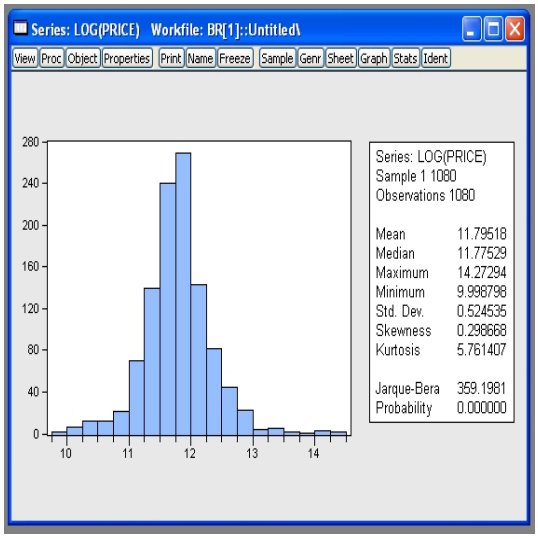


Figure 2: Statistics on log(PRICE).

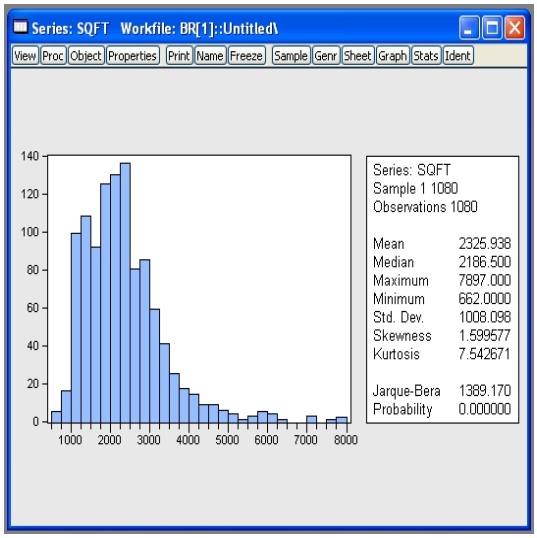


Figure 3: Statistics on SQFT.

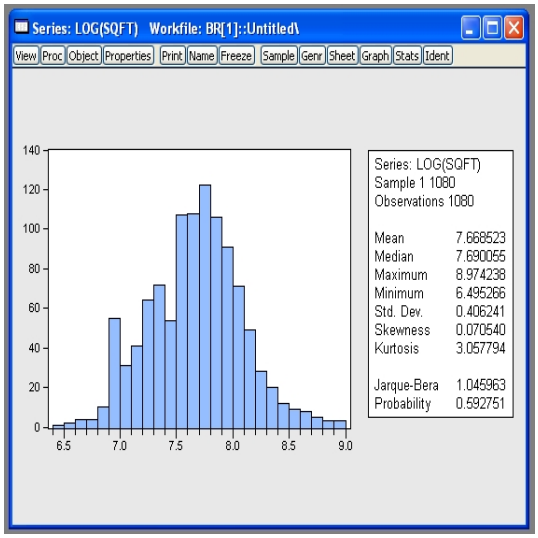


Figure 4: Statistics on log(SQFT).

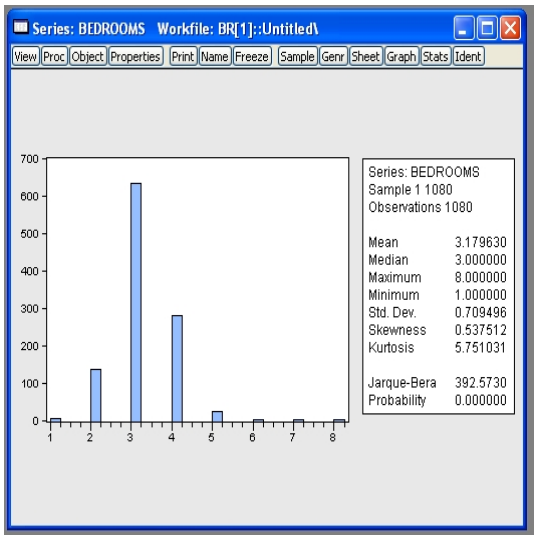


Figure 5: Statistics on BEDROOMS.

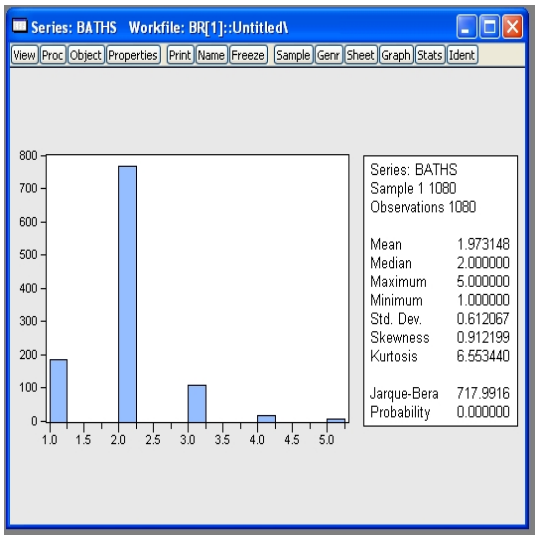


Figure 6: Statistics on BATHS.

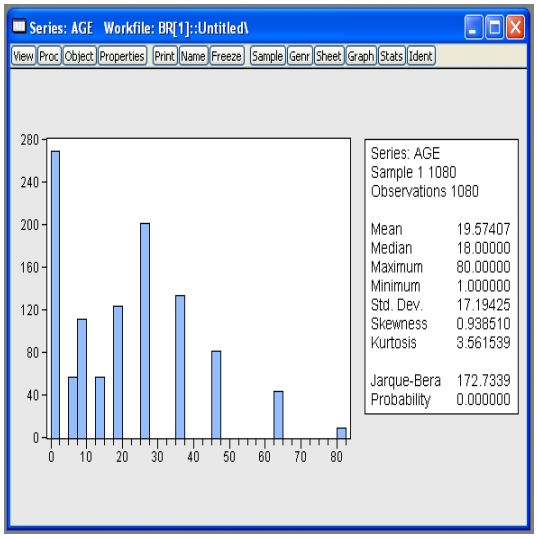


Figure 7: Statistics on AGE.

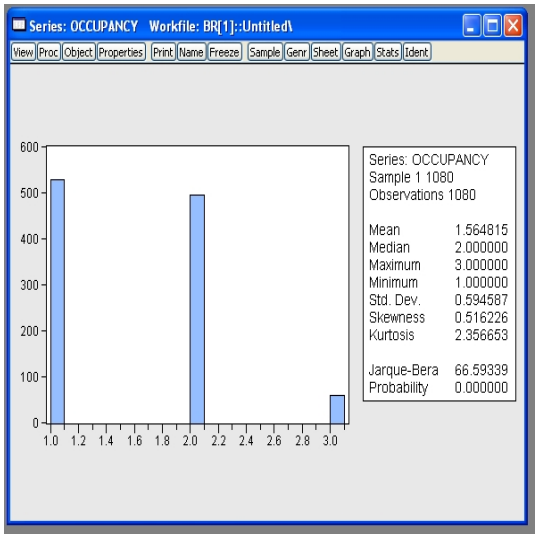


Figure 8: Statistics on OCCUPANCY.

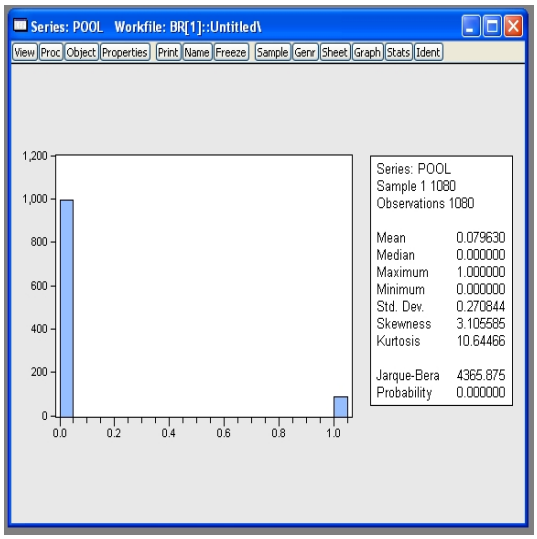


Figure 9: Statistics on POOL.

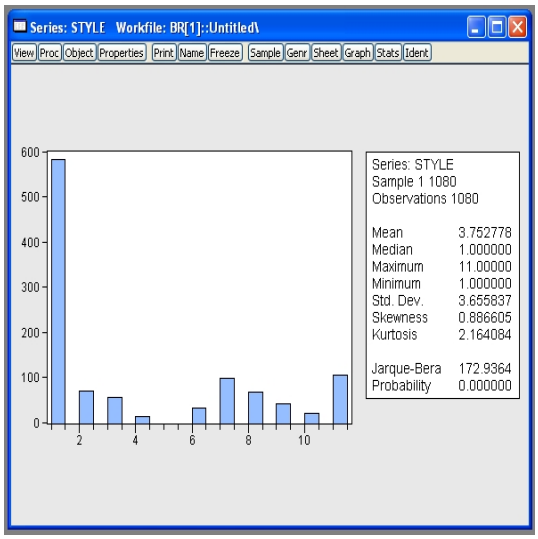


Figure 10: Statistics on STYLE.

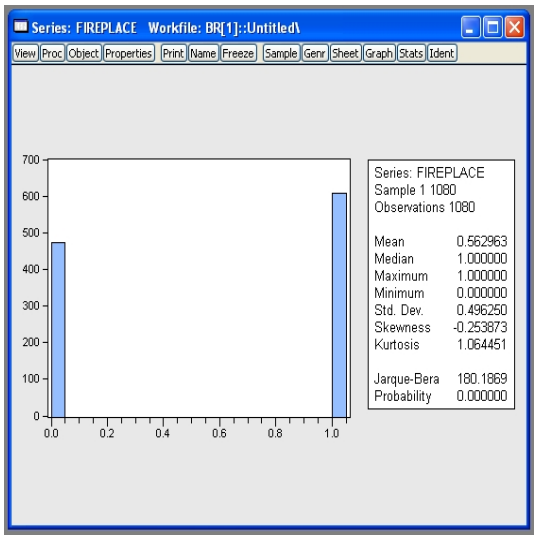


Figure 11: Statistics on FIREPLACE.

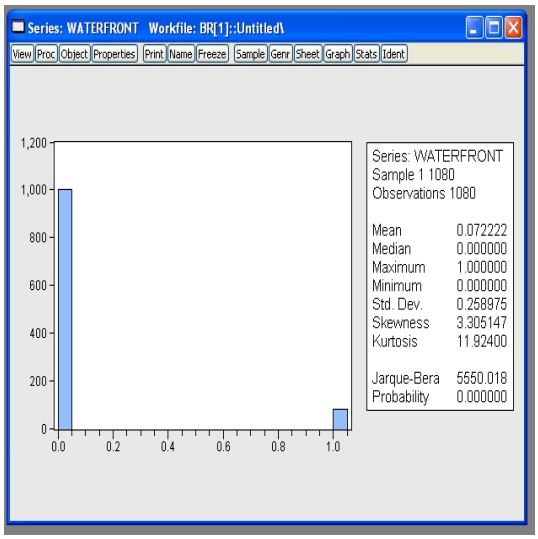


Figure 12: Statistics on WATERFRONT.

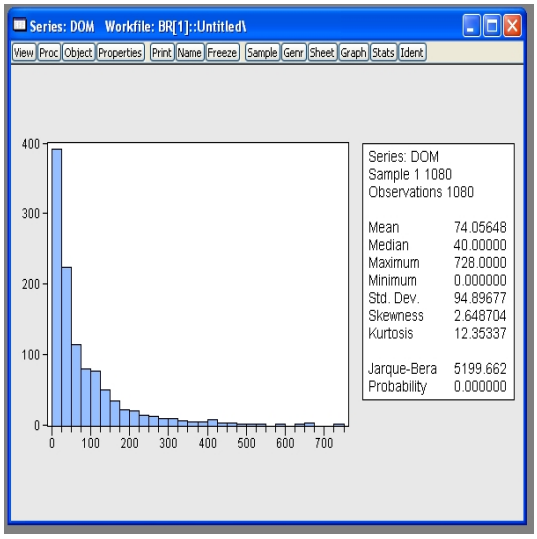


Figure 13: Statistics on DOM.

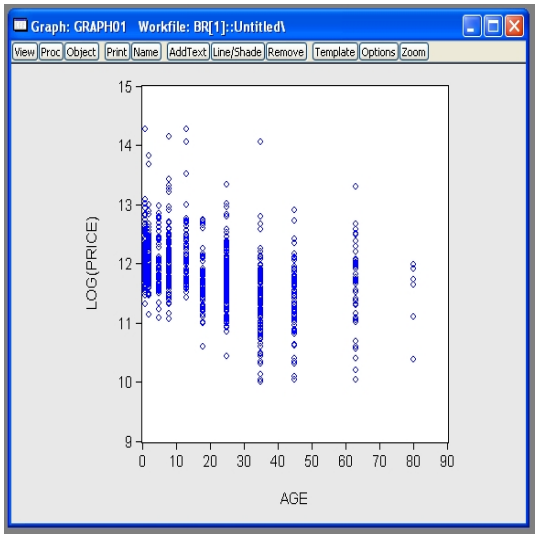


Figure 14: log(PRICE) against AGE.

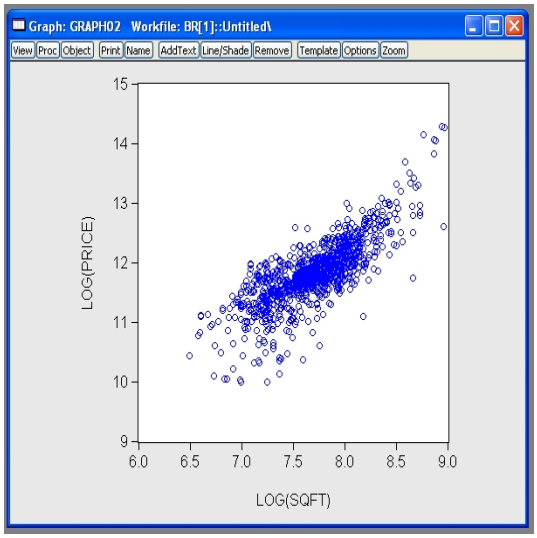


Figure 15: log(PRICE) against log(SQFT).

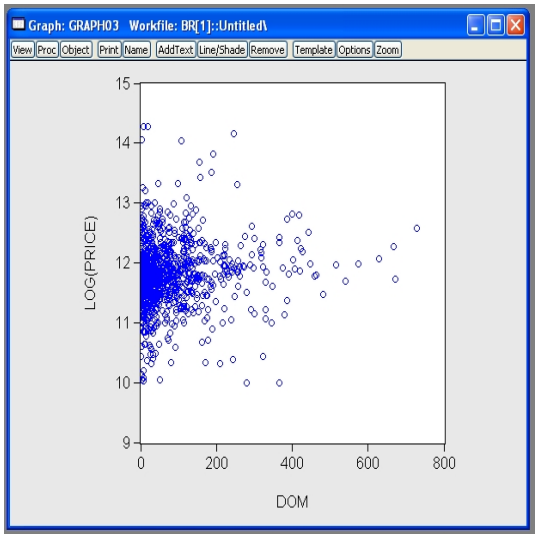


Figure 16: log(PRICE) against DOM.

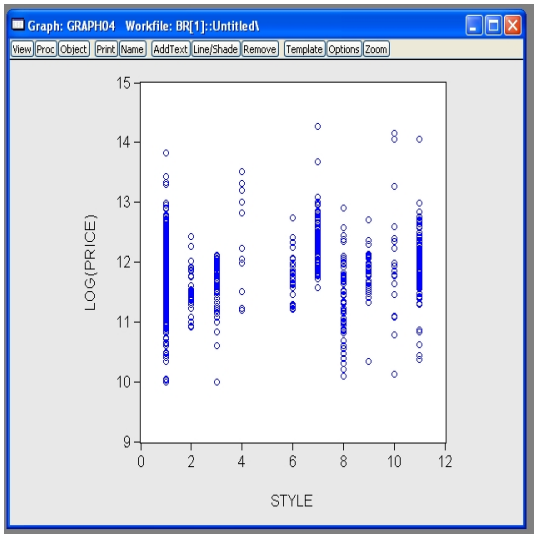


Figure 17: log(PRICE) against STYLE.

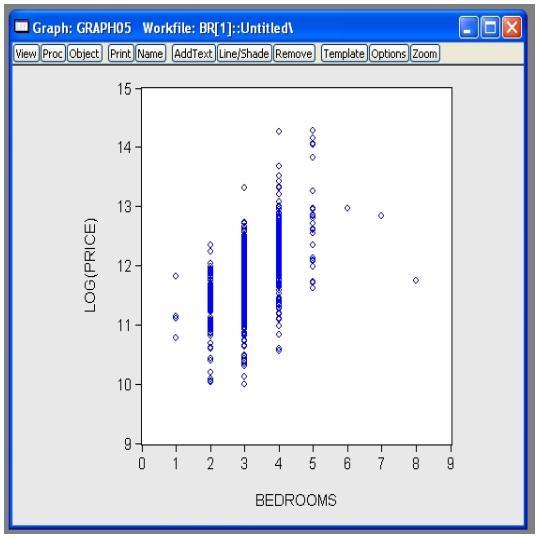


Figure 18: log(PRICE) and BEDROOMS.

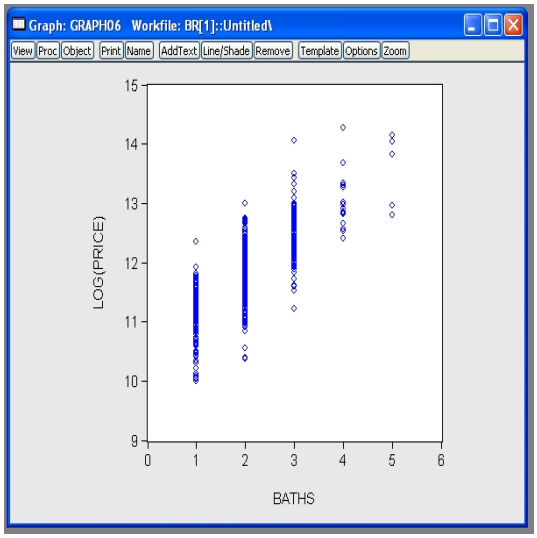


Figure 19: log(PRICE) against BATHS.

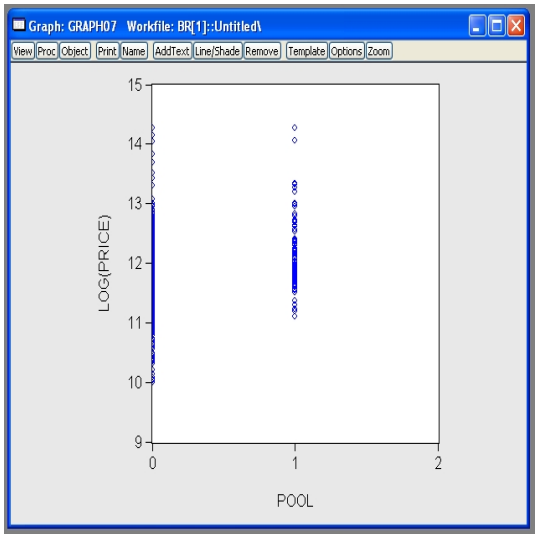


Figure 20: log(PRICE) against POOL.

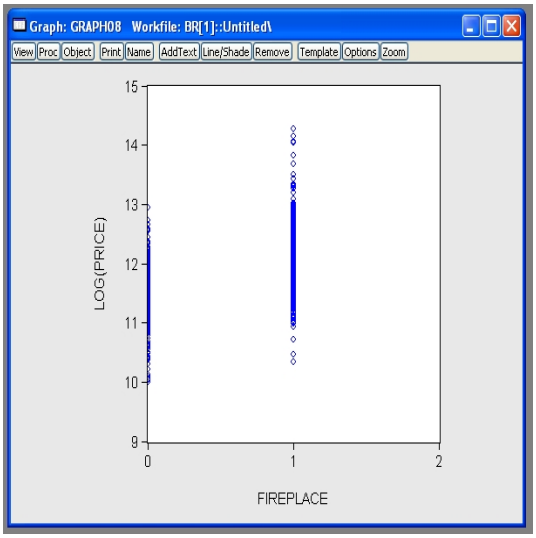


Figure 21: log(PRICE) & FIREPLACE.

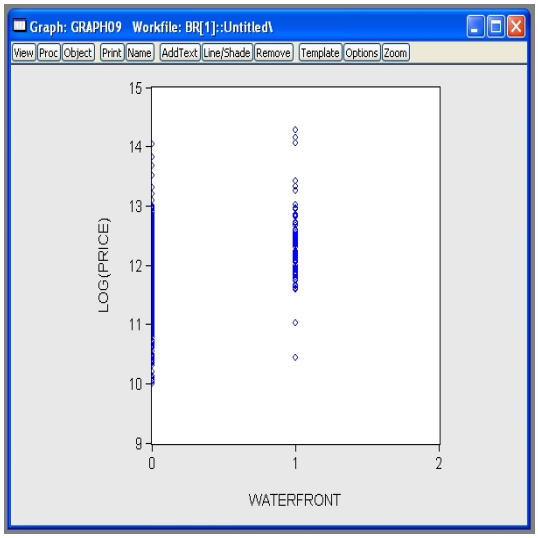
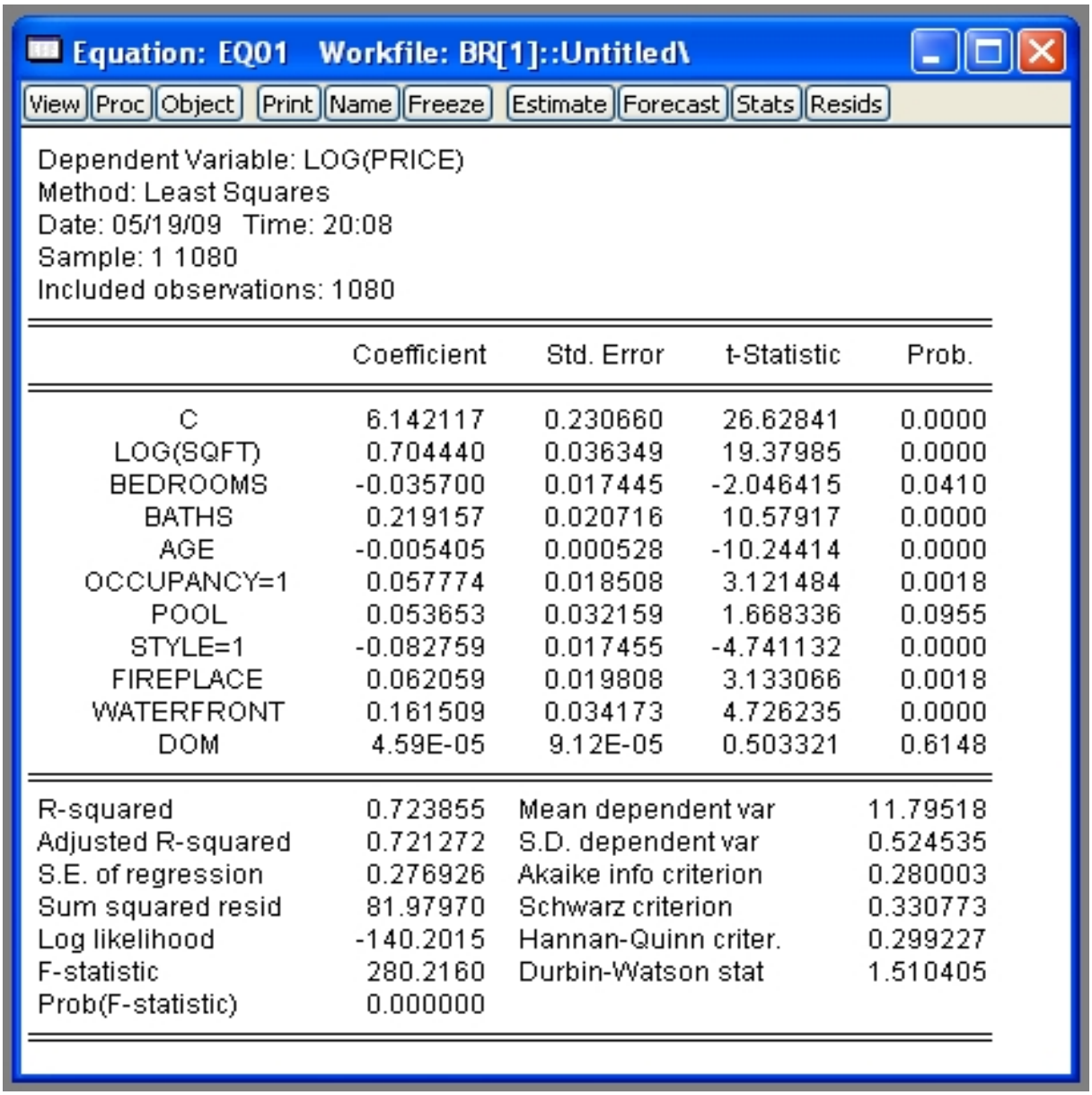


Figure 22: log(PRICE) & WATERFRONT.



Equation: EQ01 Workfile: BR[1]::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: LOG(PRICE)
Method: Least Squares
Date: 05/19/09 Time: 20:08
Sample: 1 1080
Included observations: 1080

	Coefficient	Std. Error	t-Statistic	Prob.
C	6.142117	0.230660	26.62841	0.0000
LOG(SQFT)	0.704440	0.036349	19.37985	0.0000
BEDROOMS	-0.035700	0.017445	-2.046415	0.0410
BATHS	0.219157	0.020716	10.57917	0.0000
AGE	-0.005405	0.000528	-10.24414	0.0000
OCCUPANCY=1	0.057774	0.018508	3.121484	0.0018
POOL	0.053653	0.032159	1.668336	0.0955
STYLE=1	-0.082759	0.017455	-4.741132	0.0000
FIREPLACE	0.062059	0.019808	3.133066	0.0018
WATERFRONT	0.161509	0.034173	4.726235	0.0000
DOM	4.59E-05	9.12E-05	0.503321	0.6148

R-squared	0.723855	Mean dependent var	11.79518
Adjusted R-squared	0.721272	S.D. dependent var	0.524535
S.E. of regression	0.276926	Akaike info criterion	0.280003
Sum squared resid	81.97970	Schwarz criterion	0.330773
Log likelihood	-140.2015	Hannan-Quinn criter.	0.299227
F-statistic	280.2160	Durbin-Watson stat	1.510405
Prob(F-statistic)	0.000000		

Figure 23: Regression results for `EQ01`.

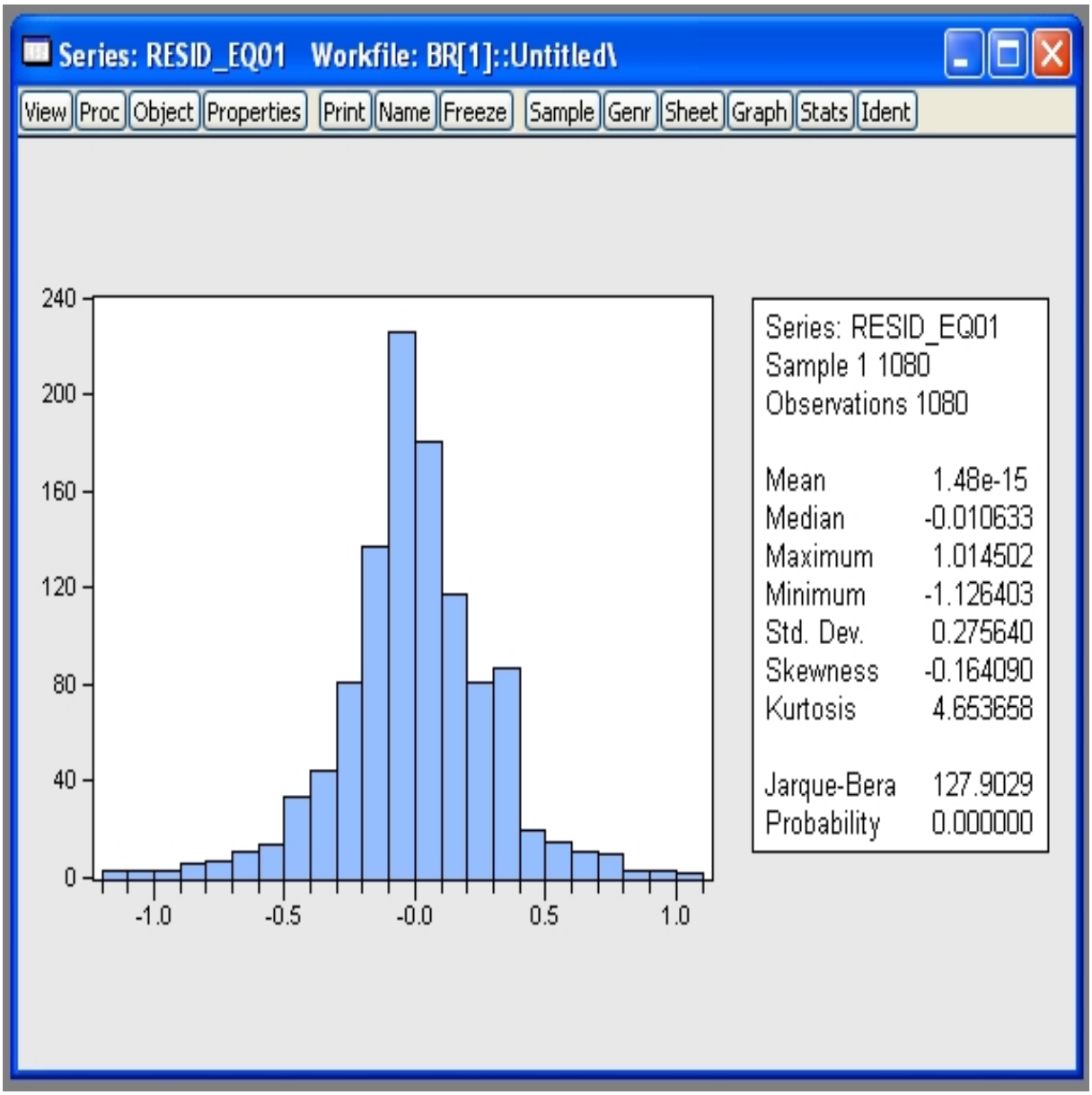


Figure 24: Descriptive statistics on the fitted residuals from `EQ01`.

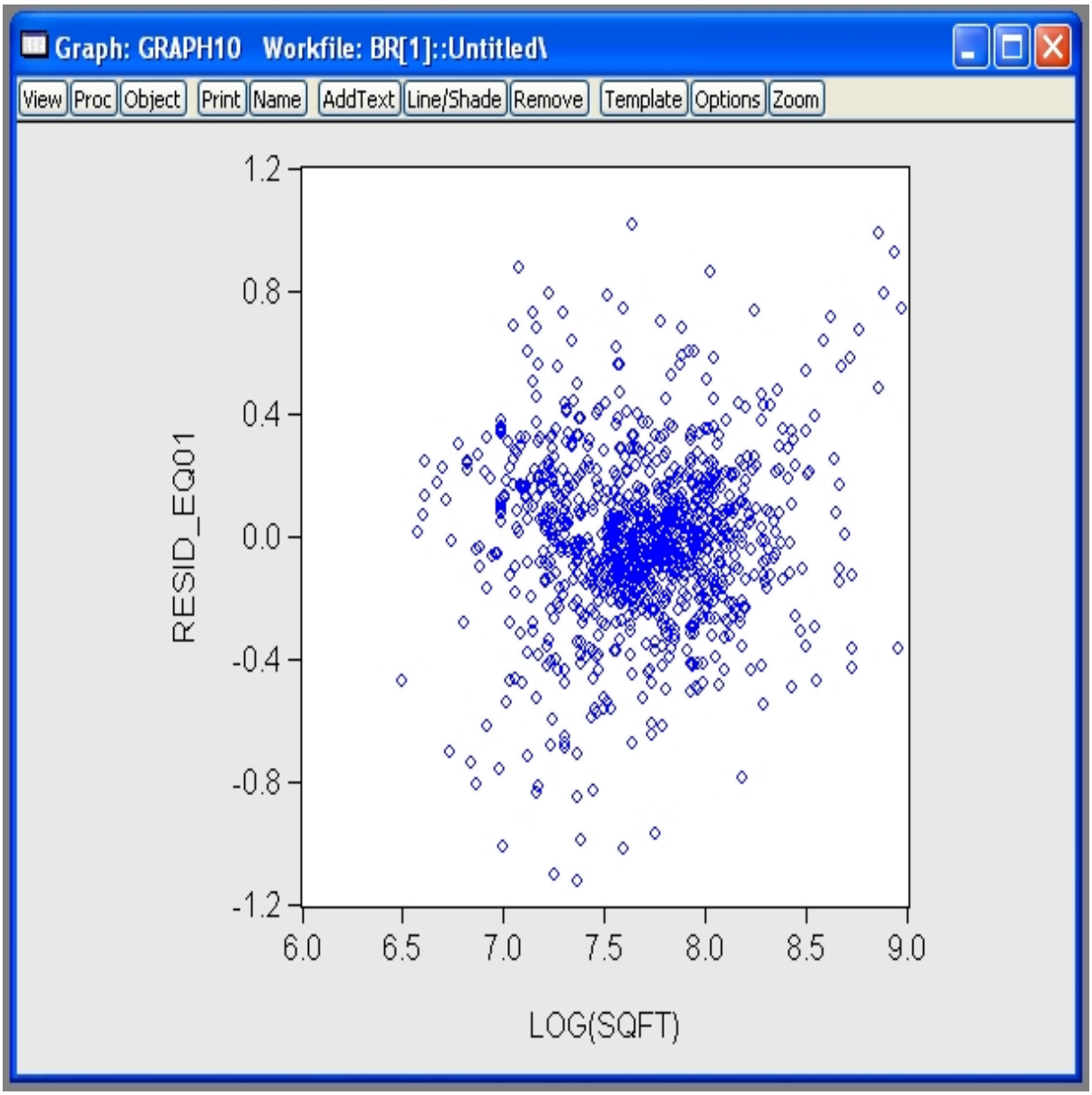


Figure 25: Scatterplot of the fitted residuals from `EQ01` against log(SQFT).

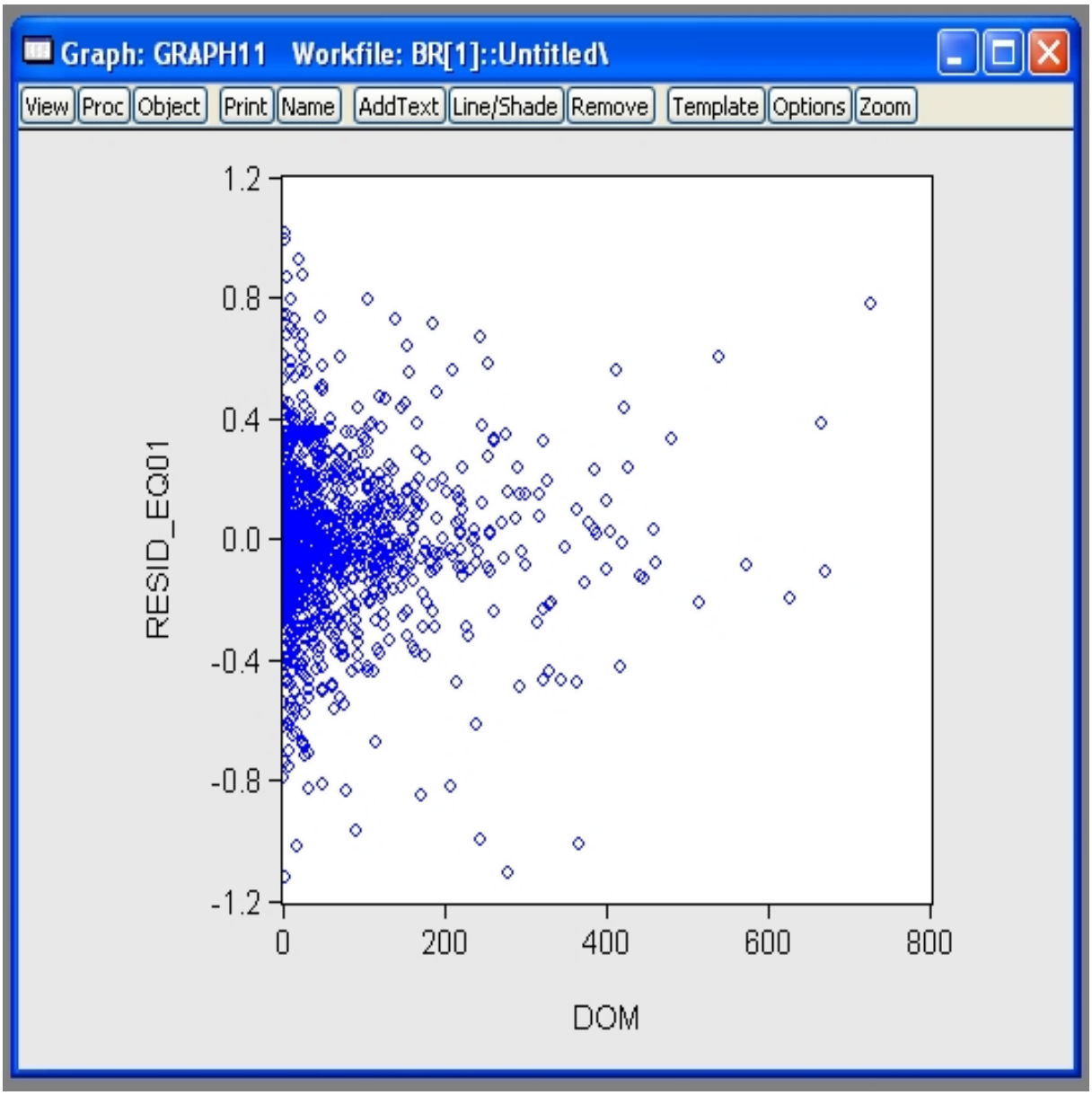


Figure 26: Scatterplot of the fitted residuals from `EQ01` against DOM.

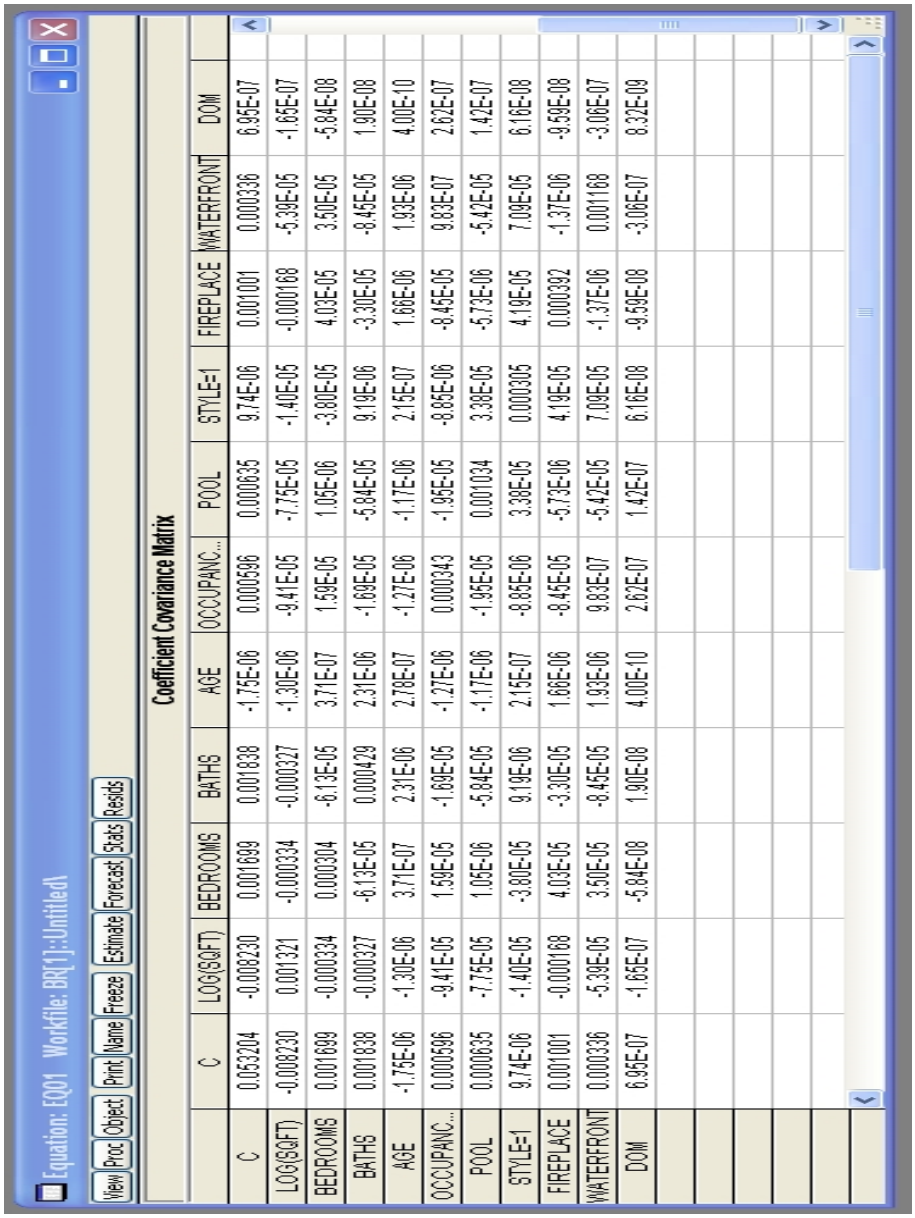


Figure 27: Estimated variance-covariance matrix of the estimated coefficients from `EQ01`.

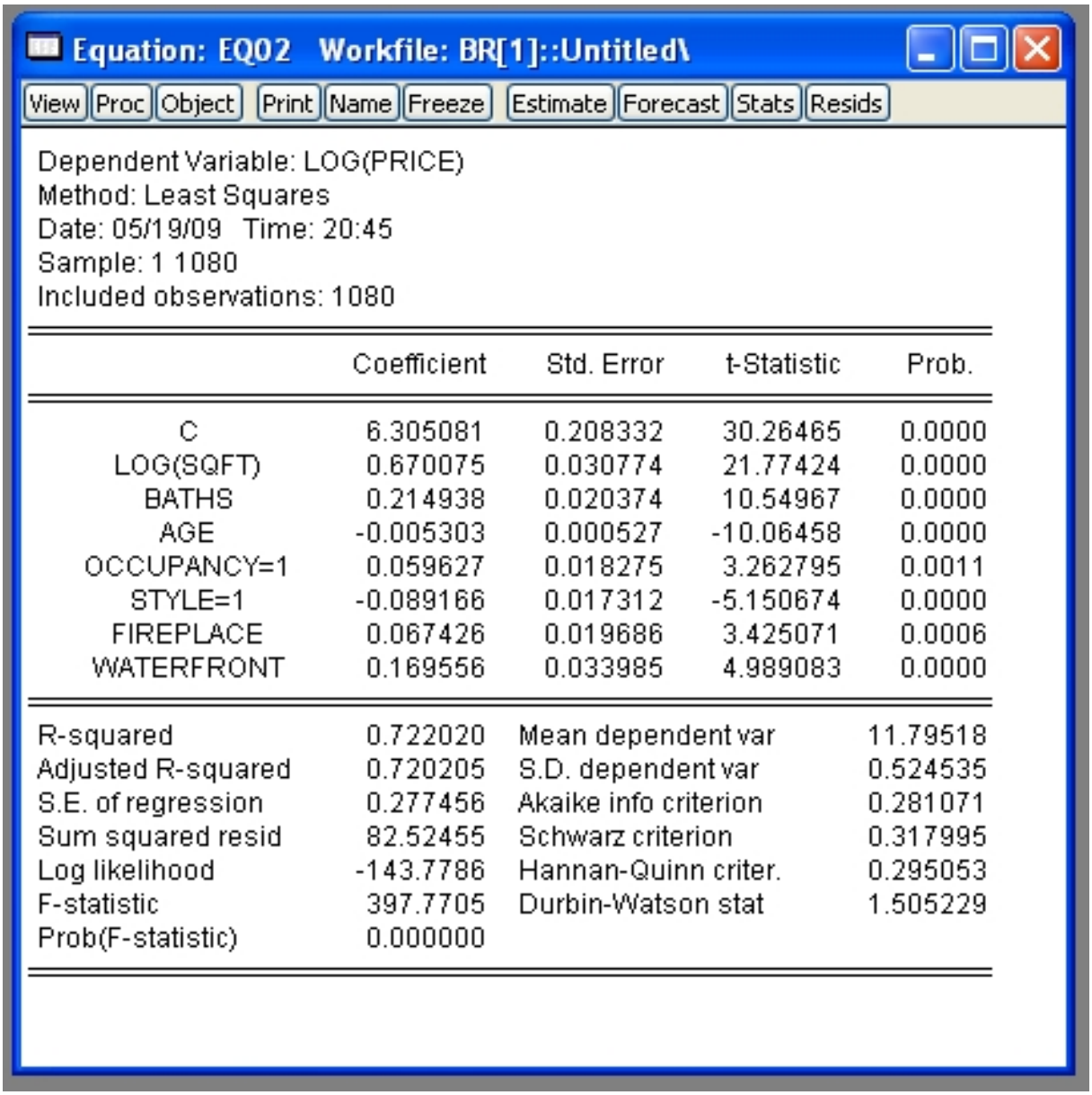
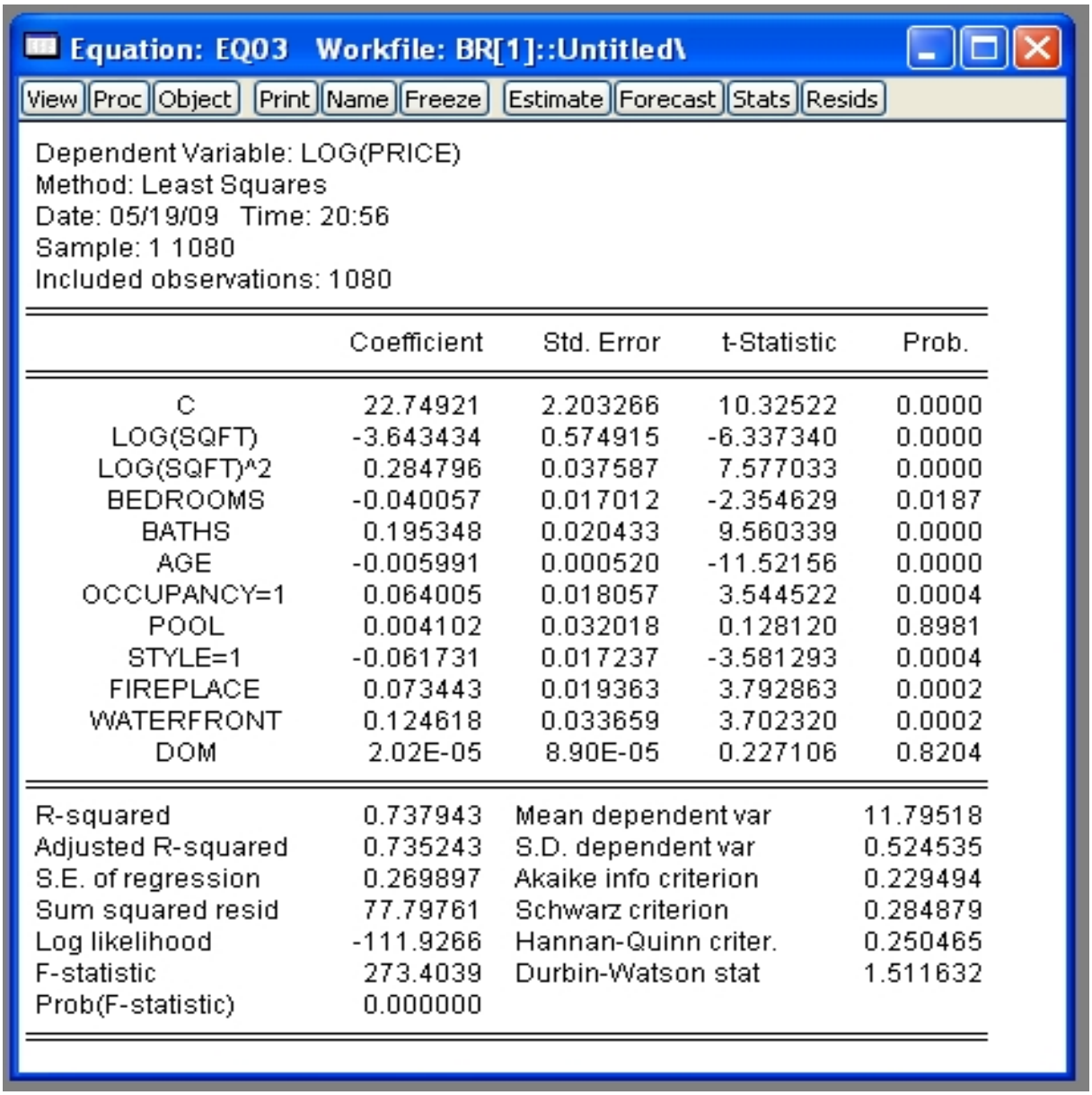


Figure 28: Regression results for `EQ02`.



Equation: EQ03 Workfile: BR[1]::Untitled\

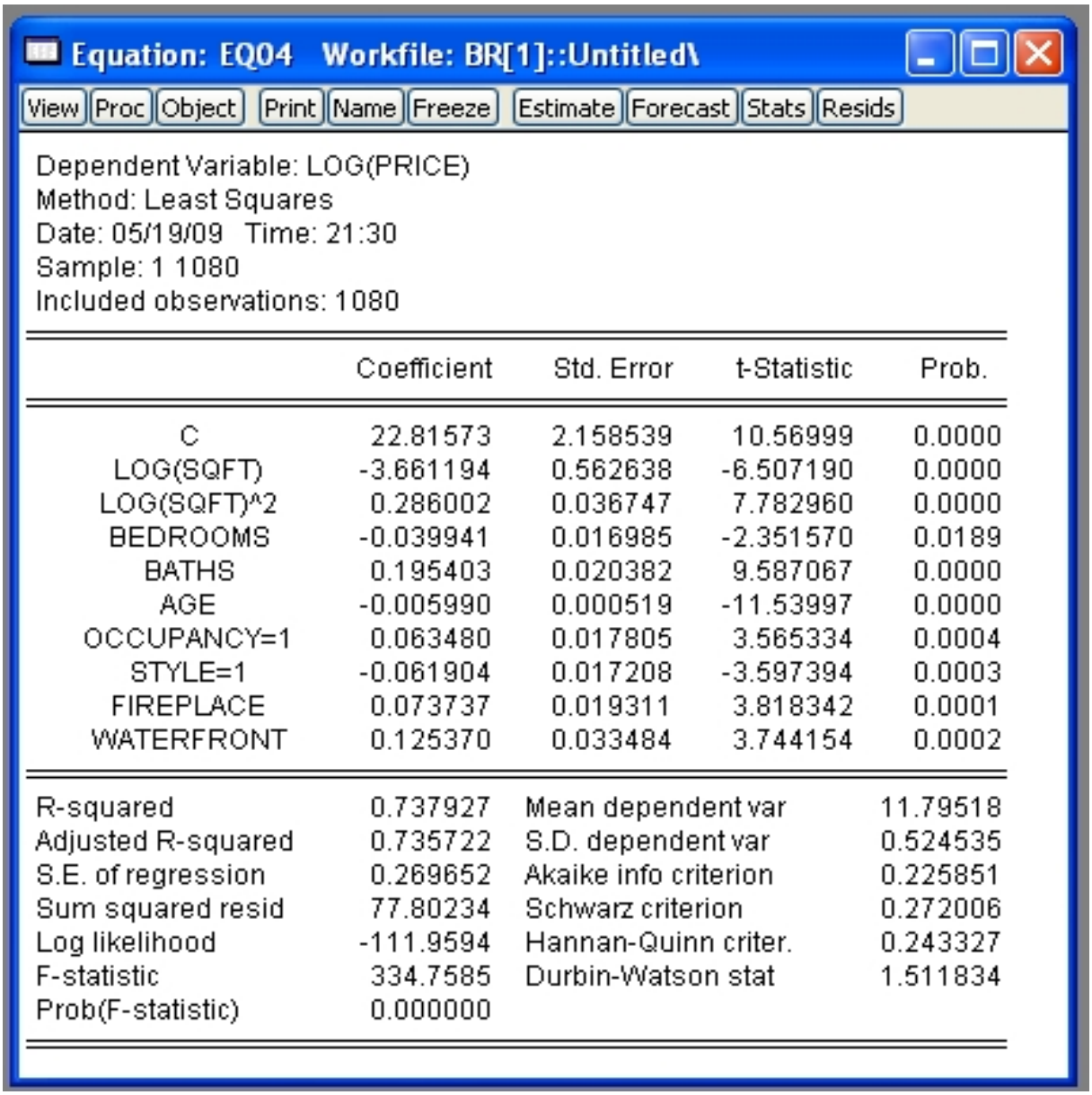
View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: LOG(PRICE)
Method: Least Squares
Date: 05/19/09 Time: 20:56
Sample: 1 1080
Included observations: 1080

	Coefficient	Std. Error	t-Statistic	Prob.
C	22.74921	2.203266	10.32522	0.0000
LOG(SQFT)	-3.643434	0.574915	-6.337340	0.0000
LOG(SQFT)^2	0.284796	0.037587	7.577033	0.0000
BEDROOMS	-0.040057	0.017012	-2.354629	0.0187
BATHS	0.195348	0.020433	9.560339	0.0000
AGE	-0.005991	0.000520	-11.52156	0.0000
OCCUPANCY=1	0.064005	0.018057	3.544522	0.0004
POOL	0.004102	0.032018	0.128120	0.8981
STYLE=1	-0.061731	0.017237	-3.581293	0.0004
FIREPLACE	0.073443	0.019363	3.792863	0.0002
WATERFRONT	0.124618	0.033659	3.702320	0.0002
DOM	2.02E-05	8.90E-05	0.227106	0.8204

R-squared	0.737943	Mean dependent var	11.79518
Adjusted R-squared	0.735243	S.D. dependent var	0.524535
S.E. of regression	0.269897	Akaike info criterion	0.229494
Sum squared resid	77.79761	Schwarz criterion	0.284879
Log likelihood	-111.9266	Hannan-Quinn criter.	0.250465
F-statistic	273.4039	Durbin-Watson stat	1.511632
Prob(F-statistic)	0.000000		

Figure 29: Regression results for `EQ03`.



Equation: EQ04 Workfile: BR[1]::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: LOG(PRICE)
Method: Least Squares
Date: 05/19/09 Time: 21:30
Sample: 1 1080
Included observations: 1080

	Coefficient	Std. Error	t-Statistic	Prob.
C	22.81573	2.158539	10.56999	0.0000
LOG(SQFT)	-3.661194	0.562638	-6.507190	0.0000
LOG(SQFT)^2	0.286002	0.036747	7.782960	0.0000
BEDROOMS	-0.039941	0.016985	-2.351570	0.0189
BATHS	0.195403	0.020382	9.587067	0.0000
AGE	-0.005990	0.000519	-11.53997	0.0000
OCCUPANCY=1	0.063480	0.017805	3.565334	0.0004
STYLE=1	-0.061904	0.017208	-3.597394	0.0003
FIREPLACE	0.073737	0.019311	3.818342	0.0001
WATERFRONT	0.125370	0.033484	3.744154	0.0002

R-squared	0.737927	Mean dependent var	11.79518
Adjusted R-squared	0.735722	S.D. dependent var	0.524535
S.E. of regression	0.269652	Akaike info criterion	0.225851
Sum squared resid	77.80234	Schwarz criterion	0.272006
Log likelihood	-111.9594	Hannan-Quinn criter.	0.243327
F-statistic	334.7585	Durbin-Watson stat	1.511834
Prob(F-statistic)	0.000000		

Figure 30: Regression results for `EQ04`.

Areas Under the Normal Curve

Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p
0.00	0.5000	0.5000	0.40	0.6554	0.3446	0.80	0.7881	0.2119	1.20	0.8849	0.1151	1.60	0.9452	0.0548
0.01	0.5040	0.4960	0.41	0.6591	0.3409	0.81	0.7910	0.2090	1.21	0.8869	0.1131	1.61	0.9463	0.0537
0.02	0.5080	0.4920	0.42	0.6628	0.3372	0.82	0.7939	0.2061	1.22	0.8888	0.1112	1.62	0.9474	0.0526
0.03	0.5120	0.4880	0.43	0.6664	0.3336	0.83	0.7967	0.2033	1.23	0.8907	0.1093	1.63	0.9484	0.0516
0.04	0.5160	0.4840	0.44	0.6700	0.3300	0.84	0.7995	0.2005	1.24	0.8925	0.1075	1.64	0.9495	0.0505
0.05	0.5199	0.4801	0.45	0.6736	0.3264	0.85	0.8023	0.1977	1.25	0.8944	0.1056	**1.65**	**0.9505**	**0.0495**
0.06	0.5239	0.4761	0.46	0.6772	0.3228	0.86	0.8051	0.1949	1.26	0.8962	0.1038	1.66	0.9515	0.0485
0.07	0.5279	0.4721	0.47	0.6808	0.3192	0.87	0.8078	0.1922	1.27	0.8980	0.1020	1.67	0.9525	0.0475
0.08	0.5319	0.4681	0.48	0.6844	0.3156	0.88	0.8106	0.1894	1.28	0.8997	0.1003	1.68	0.9535	0.0465
0.09	0.5359	0.4641	0.49	0.6879	0.3121	0.89	0.8133	0.1867	1.29	0.9015	0.0985	1.69	0.9545	0.0455
0.10	0.5398	0.4602	0.50	0.6915	0.3085	0.90	0.8159	0.1841	1.30	0.9032	0.0968	1.70	0.9554	0.0446
0.11	0.5438	0.4562	0.51	0.6950	0.3050	0.91	0.8186	0.1814	1.31	0.9049	0.0951	1.71	0.9564	0.0436
0.12	0.5478	0.4522	0.52	0.6985	0.3015	0.92	0.8212	0.1788	1.32	0.9066	0.0934	1.72	0.9573	0.0427
0.13	0.5517	0.4483	0.53	0.7019	0.2981	0.93	0.8238	0.1762	1.33	0.9082	0.0918	1.73	0.9582	0.0418
0.14	0.5557	0.4443	0.54	0.7054	0.2946	0.94	0.8264	0.1736	1.34	0.9099	0.0901	1.74	0.9591	0.0409
0.15	0.5596	0.4404	0.55	0.7088	0.2912	0.95	0.8289	0.1711	1.35	0.9115	0.0885	1.75	0.9599	0.0401
0.16	0.5636	0.4364	0.56	0.7123	0.2877	0.96	0.8315	0.1685	1.36	0.9131	0.0869	1.76	0.9608	0.0392
0.17	0.5675	0.4325	0.57	0.7157	0.2843	0.97	0.8340	0.1660	1.37	0.9147	0.0853	1.77	0.9616	0.0384
0.18	0.5714	0.4286	0.58	0.7190	0.2810	0.98	0.8365	0.1635	1.38	0.9162	0.0838	1.78	0.9625	0.0375
0.19	0.5753	0.4247	0.59	0.7224	0.2776	0.99	0.8389	0.1611	1.39	0.9177	0.0823	1.79	0.9633	0.0367
0.20	0.5793	0.4207	0.60	0.7257	0.2743	1.00	0.8413	0.1587	1.40	0.9192	0.0808	1.80	0.9641	0.0359
0.21	0.5832	0.4168	0.61	0.7291	0.2709	1.01	0.8438	0.1562	1.41	0.9207	0.0793	1.81	0.9649	0.0351
0.22	0.5871	0.4129	0.62	0.7324	0.2676	1.02	0.8461	0.1539	1.42	0.9222	0.0778	1.82	0.9656	0.0344
0.23	0.5910	0.4090	0.63	0.7357	0.2643	1.03	0.8485	0.1515	1.43	0.9236	0.0764	1.83	0.9664	0.0336
0.24	0.5948	0.4052	0.64	0.7389	0.2611	1.04	0.8508	0.1492	1.44	0.9251	0.0749	1.84	0.9671	0.0329
0.25	0.5987	0.4013	0.65	0.7422	0.2578	1.05	0.8531	0.1469	1.45	0.9265	0.0735	1.85	0.9678	0.0322
0.26	0.6026	0.3974	0.66	0.7454	0.2546	1.06	0.8554	0.1446	1.46	0.9279	0.0721	1.86	0.9686	0.0314
0.27	0.6064	0.3936	0.67	0.7486	0.2514	1.07	0.8577	0.1423	1.47	0.9292	0.0708	1.87	0.9693	0.0307
0.28	0.6103	0.3897	0.68	0.7517	0.2483	1.08	0.8599	0.1401	1.48	0.9306	0.0694	1.88	0.9699	0.0301
0.29	0.6141	0.3859	0.69	0.7549	0.2451	1.09	0.8621	0.1379	1.49	0.9319	0.0681	1.89	0.9706	0.0294
0.30	0.6179	0.3821	0.70	0.7580	0.2420	1.10	0.8643	0.1357	1.50	0.9332	0.0668	1.90	0.9713	0.0287
0.31	0.6217	0.3783	0.71	0.7611	0.2389	1.11	0.8665	0.1335	1.51	0.9345	0.0655	1.91	0.9719	0.0281
0.32	0.6255	0.3745	0.72	0.7642	0.2358	1.12	0.8686	0.1314	1.52	0.9357	0.0643	1.92	0.9726	0.0274
0.33	0.6293	0.3707	0.73	0.7673	0.2327	1.13	0.8708	0.1292	1.53	0.9370	0.0630	1.93	0.9732	0.0268
0.34	0.6331	0.3669	0.74	0.7704	0.2296	1.14	0.8729	0.1271	1.54	0.9382	0.0618	1.94	0.9738	0.0262
0.35	0.6368	0.3632	0.75	0.7734	0.2266	1.15	0.8749	0.1251	1.55	0.9394	0.0606	1.95	0.9744	0.0256
0.36	0.6406	0.3594	0.76	0.7764	0.2236	1.16	0.8770	0.1230	1.56	0.9406	0.0594	**1.96**	**0.9750**	**0.0250**
0.37	0.6443	0.3557	0.77	0.7794	0.2206	1.17	0.8790	0.1210	1.57	0.9418	0.0582	1.97	0.9756	0.0244
0.38	0.6480	0.3520	0.78	0.7823	0.2177	1.18	0.8810	0.1190	1.58	0.9429	0.0571	1.98	0.9761	0.0239
0.39	0.6517	0.3483	0.79	0.7852	0.2148	1.19	0.8830	0.1170	1.59	0.9441	0.0559	1.99	0.9767	0.0233

Figure 31: Statistical table for $N(0,1)$. These tables are taken from `http://fsweb.berry.edu/academic/education/vbissonnette/tables/tables.html`

Critical Values of the t Distribution

df	2-tailed testing			1-tailed testing		
	0.1	0.05	0.01	0.1	0.05	0.01
5	2.015	2.571	4.032	1.476	2.015	3.365
6	1.943	2.447	3.707	1.440	1.943	3.143
7	1.895	2.365	3.499	1.415	1.895	2.998
8	1.860	2.306	3.355	1.397	1.860	2.896
9	1.833	2.262	3.250	1.383	1.833	2.821
10	1.812	2.228	3.169	1.372	1.812	2.764
11	1.796	2.201	3.106	1.363	1.796	2.718
12	1.782	2.179	3.055	1.356	1.782	2.681
13	1.771	2.160	3.012	1.350	1.771	2.650
14	1.761	2.145	2.977	1.345	1.761	2.624
15	1.753	2.131	2.947	1.341	1.753	2.602
16	1.746	2.120	2.921	1.337	1.746	2.583
17	1.740	2.110	2.898	1.333	1.740	2.567
18	1.734	2.101	2.878	1.330	1.734	2.552
19	1.729	2.093	2.861	1.328	1.729	2.539
20	1.725	2.086	2.845	1.325	1.725	2.528
21	1.721	2.080	2.831	1.323	1.721	2.518
22	1.717	2.074	2.819	1.321	1.717	2.508
23	1.714	2.069	2.807	1.319	1.714	2.500
24	1.711	2.064	2.797	1.318	1.711	2.492
25	1.708	2.060	2.787	1.316	1.708	2.485
26	1.706	2.056	2.779	1.315	1.706	2.479
27	1.703	2.052	2.771	1.314	1.703	2.473
28	1.701	2.048	2.763	1.313	1.701	2.467
29	1.699	2.045	2.756	1.311	1.699	2.462
30	1.697	2.042	2.750	1.310	1.697	2.457
40	1.684	2.021	2.704	1.303	1.684	2.423
50	1.676	2.009	2.678	1.299	1.676	2.403
60	1.671	2.000	2.660	1.296	1.671	2.390
80	1.664	1.990	2.639	1.292	1.664	2.374
100	1.660	1.984	2.626	1.290	1.660	2.364
120	1.658	1.980	2.617	1.289	1.658	2.358
∞	1.645	1.960	2.576	1.282	1.645	2.327

Figure 32: Statistical table for Student's $t(r)$.

Critical Values of the F Distribution
($\alpha = .05$)

df within	df between										
	1	2	3	4	5	6	7	8	12	24	∞
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.68	4.53	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.79	2.61	2.41
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69	2.51	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.20	2.01	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.18	1.98	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.16	1.96	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.12	1.91	1.66
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.09	1.89	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.00	1.79	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.92	1.70	1.39
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	1.88	1.65	1.33
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.85	1.63	1.28
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.83	1.61	1.26
∞	3.84	3.00	2.61	2.37	2.22	2.10	2.01	1.94	1.75	1.52	1.00

Figure 33: Statistical table for $F(m, p)$ at the 5% level.

Critical Values of the F Distribution
($\alpha = .01$)

df within	df between										
	1	2	3	4	5	6	7	8	12	24	∞
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	9.89	9.47	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.72	7.31	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.47	6.07	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.67	5.28	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.11	4.73	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.71	4.33	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.40	4.02	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.16	3.78	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	3.96	3.59	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.80	3.43	3.01
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.67	3.29	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.55	3.18	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.46	3.08	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.37	3.00	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.30	2.92	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.23	2.86	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.17	2.80	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.12	2.75	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.07	2.70	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.03	2.66	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	2.99	2.62	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	2.96	2.58	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	2.93	2.55	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	2.90	2.52	2.07
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	2.87	2.49	2.04
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.84	2.47	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.66	2.29	1.81
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.50	2.12	1.60
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.42	2.03	1.50
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.37	1.98	1.43
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.34	1.95	1.38
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.19	1.79	1.00

Figure 34: Statistical table for $F(m, p)$ at the 1% level.

Critical Values of the χ^2 Distribution

df	Area in the Upper Tail					
	0.99	0.95	0.9	0.1	**0.05**	**0.01**
1	0.000	0.004	0.016	2.706	3.841	6.635
2	0.020	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.345
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475
8	1.646	2.733	3.490	13.362	15.507	20.090
9	2.088	3.325	4.168	14.684	16.919	21.666
10	2.558	3.940	4.865	15.987	18.307	23.209
11	3.053	4.575	5.578	17.275	19.675	24.725
12	3.571	5.226	6.304	18.549	21.026	26.217
13	4.107	5.892	7.042	19.812	22.362	27.688
14	4.660	6.571	7.790	21.064	23.685	29.141
15	5.229	7.261	8.547	22.307	24.996	30.578
16	5.812	7.962	9.312	23.542	26.296	32.000
17	6.408	8.672	10.085	24.769	27.587	33.409
18	7.015	9.390	10.865	25.989	28.869	34.805
19	7.633	10.117	11.651	27.204	30.144	36.191
20	8.260	10.851	12.443	28.412	31.410	37.566
21	8.897	11.591	13.240	29.615	32.671	38.932
22	9.542	12.338	14.041	30.813	33.924	40.289
23	10.196	13.091	14.848	32.007	35.172	41.638
24	10.856	13.848	15.659	33.196	36.415	42.980
25	11.524	14.611	16.473	34.382	37.652	44.314

Figure 35: Statistical table for $\chi^2(q)$.